

# VIBROACOUSTIC COUPLING BETWEEN AN INTERNAL ACOUSTIC FIELD AND EACH VIBRATION OF END PLATES FOR CYLINDRICAL STRUCTURES

B.Eng. Tanaka S.<sup>1</sup>, B.Eng. Nishikawa K.<sup>1</sup>, and Prof. D.Eng. Moriyama H.<sup>2</sup>  
 Course of Mechanical Engineering, Graduate School of Tokai University, Japan <sup>1</sup>  
 Department of Prime Mover Engineering, Tokai University, Japan <sup>2</sup>

**Abstract:** This paper describes the vibroacoustic coupling between the structural vibrations and internal sound field of thin structures. In this study, a cylindrical structure with thin end plates is subjected to a harmonic point force at one of its ends. A natural frequency of the end plates is selected as the forcing frequency, causing the one end plate to vibrate in high-order mode. The resulting vibroacoustic coupling is then analyzed by theoretically and experimentally by considering the dynamic behavior of the plates and the acoustic characteristics of the internal sound field as a function of the cylinder length and plate thickness. The theoretical results are validated experimentally through an excitation experiment using an experimental apparatus that emulates the analytical model. As a result of the above consideration, the behavior of vibroacoustic coupling is characterized by the distinction with respect to whether some acoustic modes take part in its phenomena.

**Keywords:** VIBROACOUSTIC COUPLING, CYLINDRICAL STRUCTURE, THIN END PLATE, INTERNAL SOUND FIELD, PLATE VIBRATION

## 1. Introduction

In case vessels, vehicles, aerospace apparatus, etc., whose thicknesses are much thinner than the representative dimensions, are subjected to external periodic forces, the structural vibrations are coupled with the internal sound fields formed by the vibrations. Vibroacoustic coupling is caused by energy transfer between vibration and acoustic modes and its promotion depends greatly on the efficiency of the energy transfer<sup>1,2</sup>. In many studies, because the thin structure was subjected to a harmonic point force, the vibration mode caused by the eigenfrequency was hardly affected by neighboring vibration modes in the lower frequency range, so that vibroacoustic coupling was considered as phenomena based on a pair of the vibration and acoustic modes<sup>3,4</sup>. The accumulation number of the acoustic modes exceeds greatly that of the vibration modes in the higher frequency range, increasing remarkably with the excitation frequency<sup>5</sup>, with the result that vibroacoustic coupling may not be taken as phenomena based on a pair of the vibration and acoustic modes due to coupling with some acoustic modes. Of course the above studies took into consideration such a influence in their analyses but coupling phenomena with some acoustic modes was hardly mentioned in detail.

In this study, the cylindrical structure with plates at both ends, which was also taken in the above-described studies, is adopted and one end plate is excited by a harmonic point force, whose frequencies cause the plate to vibrate in high-order symmetric or unsymmetric mode. Vibroacoustic coupling between the plate vibrations and internal sound field is estimated theoretically and experimentally from the increases in the plate deflection and sound pressure level, while changing the thickness of the plate and the cylinder length. The acoustic characteristics are theoretically consider by means of the various aspects of the acoustic energy stored in the acoustic modes, the modal density of the sound field and so on. As a result, the behavior of vibroacoustic coupling is characterized by the distinction as to whether the some acoustic modes take part in coupling.

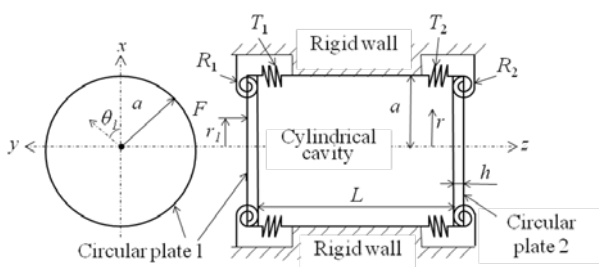


Fig. 1 Analytical model.

## 2. Analytical model

The analytical model consists of a cavity with two circular end plates, as shown in Fig. 1. The plates are supported by translational and rotational springs distributed at constant intervals and the support conditions are determined by their respective spring stiffnesses  $T_1$ ,  $T_2$ ,  $R_1$  and  $R_2$ , where the suffixes 1 and 2 indicate plates 1 and 2, respectively. The plates have a Young's modulus  $E$ , a Poisson's ratio  $\nu$  and varying thickness  $h$ . The sound field, assumed cylindrical, has the same radius as that of the plates, and varying length because the resonance frequency depends on the length. The boundary conditions are considered structurally and acoustically rigid at the lateral wall between the structure and sound field. The coordinates used are radius  $r$ , angle  $\theta$  between the planes of the plates and the cross-sectional plane of the cavity and distance  $z$  along the cylinder axis. A periodic point force  $F_1$  is applied to plate 1 at distance  $r_1$  and angle  $\theta_1$ . The natural frequency of the plate is employed as the excitation frequency.

## 3. Experimental apparatus and method

Figure 2 shows the experimental apparatus used in this study. The structure consists of a steel cylinder with circular aluminum end plates that are 3 or 4 mm thick. The cylinder has an inner radius of 153 mm, and this length can be varied from 500 to 2000 mm to emulate the analytical model. A harmonic point force excites only one end plate and its frequencies make the plate vibrate in the (1,0) and (0,1) mode. This force is applied to plate 1 by a small vibrator, whose amplitude is controlled to be 1 N. The position  $r_1$  of the point force is normalized by radius  $a$ , and is set to  $r_1/a = 0.4$  and  $0.2$  at the (1,0) and (0,1) modes, respectively. To estimate the vibroacoustic coupling characteristics, the sound pressure levels in the cavities are

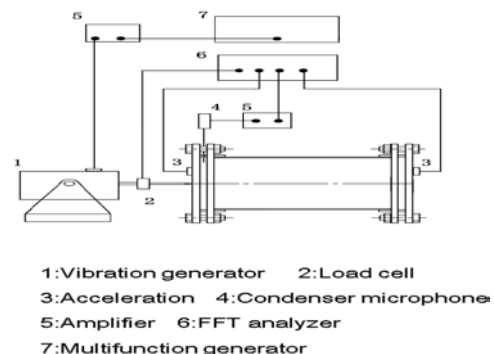


Fig. 2 Experimental apparatus.

measured using condenser microphones with a probe tube, whose tip is located near the plates and the cylinder wall. Moreover, the accelerations of the end plates are also measured using acceleration sensors installed on both plates.

## 4. Results and discussion

### 4.1 Vibroacoustic coupling with unsymmetric vibration mode

In the theoretical study, the plates are assumed to be aluminum having a Young's modulus  $E$  of 71 GPa and a Poisson's ratio  $\nu$  of 0.33. The radius  $a$  of the plates is constant at 153 mm, whereas their thickness  $h$  ranges from 2 to 4 mm. The length of the cylindrical sound field having the same radius as that of the plates varies from 100 to 2000 mm. The support conditions of the plates, which have flexural rigidity  $D = Eh^3/[12(1 - \nu^2)]$ , are expressed by the non-dimensional stiffness parameters  $T_n (= T_1 a^3/D = T_2 a^3/D)$  and  $R_n (= R_1 a/D = R_2 a/D)$ . These values are identical for both plates. If  $R_n$  ranges from 0 to  $10^8$  when  $T_n$  is  $10^8$ , the support condition can be assumed from a simple support to a clamped support. The actual condition adopts  $T_n = 10^8$   $R_n = 10^1$  to get closer to the experimental support condition. The plate and sound field eigenfrequency characteristics involved in the vibroacoustic coupling are represented by the natural frequency  $f_{nm}$  corresponding to the  $(n,m)$  mode and the resonance frequency  $f_{npq}$  corresponding to the  $(n,p,q)$  mode.

Here, the  $(1,0)$  mode is taken as a unsymmetric mode and the thicknesses  $h$  of 3 and 4 mm are chosen. Figure 3 shows the sound pressure level  $L_{pv}$  averaged over the entire sound field as a function of the cylinder length  $L$ . To demonstrate the theoretical results, the sound pressure level  $L_p$  measured in the experiment is also plotted in the same figure.  $L_{pv}$  at  $h = 4$  mm has peaks at  $L = 770, 1160, 1540$ , and  $1930$  mm, while  $L_{pv}$  at  $h = 3$  mm tends to decrease in a steadier manner with increasing  $L$ .  $L_p$  at  $h = 4$  mm also shows peaks in the vicinities of  $L = 825$  and  $1200$  mm, although the peaks become unclear in the range of a longer cylinder length, and then

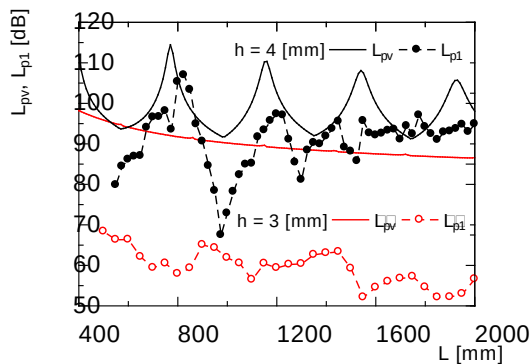


Fig. 3 Comparison between theoretical and experimental results for sound pressure level.

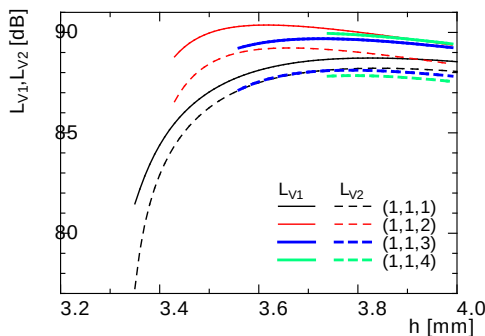


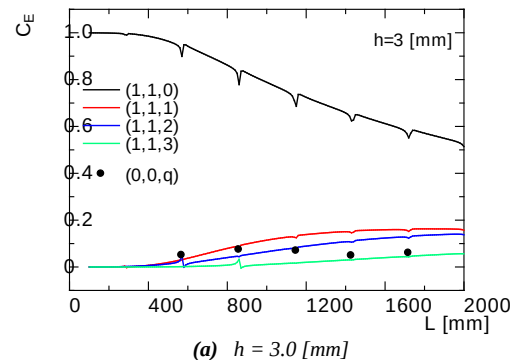
Fig. 5 Variation in plate vibration level with plate thickness in  $(1,0)$  mode.

$L_p$  at  $h = 3$  mm decreases gradually with increasing  $L$  and does not have peaks like  $L_p$  at  $h = 4$  mm. The experimental results reveal that the theoretical estimation of acoustic characteristics is reasonable as to whether the peaks appear. In terms of such a peak appearance, the contribution  $C_E$  can be defined as the ratio of the acoustic energy  $E_{npq}$  stored in the specific  $(n,p,q)$  mode to the total acoustic energy  $E_{all}$  of the entire sound field:

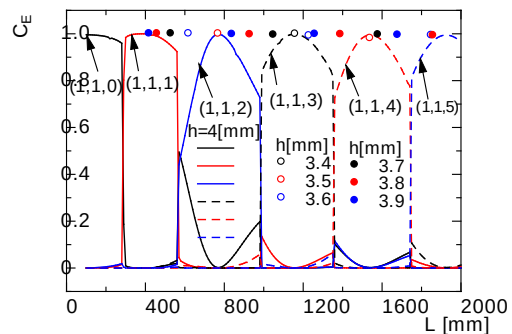
$$C_E = E_{npq} / E_{all}. \quad (1)$$

Figures 4(a) and 4(b) show  $C_E$  for  $(1,1,q)$  modes as a function of  $L$ . In Fig. 4(a),  $C_E$  is taken as the contribution at  $h = 3$  mm, at which  $L_{pv}$  and  $L_p$  did not have obvious peaks, and the longitudinal order  $q$  of the  $(0,0,q)$  mode ranges from 2 to 6.  $C_E$  of the  $(1,1,0)$  mode decreases gradually with increasing  $L$ , being close to 1.0 in the short  $L$  range;  $C_E$  of the other  $(1,1,q)$  modes are suppressed in the entire range of  $L$ . Here, besides the  $(1,1,q)$  modes,  $C_E$  of the  $(0,0,q)$  modes is also plotted at lengths for which  $f_{00q}$  is equal to  $f_{10}$ .  $C_E$  of the  $(1,1,0)$  mode decreases abruptly, which is the reason why the  $(0,0,q)$  modes contribute to the formation of the sound field. In this case,  $f_{10}$  never reaches  $f_{11q}$  over the whole range of  $L$ , with the result that  $C_E$  of  $(1,1,q)$  modes is suppressed owing to weakened coupling between the  $(1,0)$  and  $(1,1,q)$  modes. In contrast with the  $(1,1,q)$  modes,  $f_{10}$  reaches  $f_{00q}$  at the some lengths, whereas  $C_E$  of  $(0,0,q)$  modes is also suppressed because of the large difference between the  $(1,0)$  and  $(0,0,q)$  modes in modal shapes.

In Fig. 4(b),  $C_E$  of  $h = 4$  mm, for which  $f_{10}$  can reach  $f_{11q}$ , is indicated by lines corresponding to each  $(1,1,q)$  mode. The range of  $L$  increasing  $C_E$  shifts with  $q$ , in particular,  $L_{pv}$  is maximized as  $C_E$  approaches 1.0. The other  $C_E$ , under the condition that  $f_{10}$  can reach  $f_{11q}$ , is also plotted by means of changing  $h$  as  $L_{pv}$  becomes peaks that occur at an integer  $q$  starting from 1 with increasing  $L$ . These values are close to unity as well as those at  $h = 4$  mm. Such a peak appearance is obviously caused by the promotion of vibroacoustic coupling between the  $(1,0)$  and  $(1,1,q)$  modes, which depends on not only the closeness of the eigenfrequencies but also the similarity of the modal shapes between the vibration and acoustic systems.



(a)  $h = 3.0$  [mm]



(b)  $h = 3.4 - 4.0$  [mm]

Fig. 4 Contribution of each acoustic mode to formation of sound field as function of cylinder length in  $(1,0)$  mode.

To grasp the behavior of the plate vibration under the condition that coupling is promoted, Fig. 5 shows the vibration levels  $L_{v1}$  and  $L_{v2}$  for quadratic velocities averaged on plates 1 and 2, respectively, as a function of  $h$ , for which  $L$  to adjust  $f_{11q}$  to  $f_{10}$  is chosen. Not plotted up to approximately  $h = 3.3$  mm because  $f_{10}$  cannot reach  $f_{11q}$ , subsequently,  $L_{v1}$  and  $L_{v2}$  appear in the (1,1,1) mode and increase abruptly with  $h$ , and then the increase tendency is weakened and changes into a gradual decrease tendency. As the value of  $q$  increases,  $L_{v1}$  and  $L_{v2}$  appear from the larger values of  $h$  and their difference is expanded. However,  $L_{v1}$  always exceeds  $L_{v2}$  in the range of  $h$ , in which they appear.

Figure 6 shows  $L_{v1}$  and  $L_{v2}$  at  $h = 4$  mm as a function of  $L$ , and the experimental accelerations  $\alpha_1$  and  $\alpha_2$  for the vibrations of plate 1 and 2 are plotted to be compared with the theoretical results.  $L_{v1}$  remains more than an arbitrary level in the entire range of  $L$ , having somewhat large variations, whereas  $L_{v2}$  tends to decrease in addition to relatively small variations with increasing  $L$ . The difference between these levels expands with increasing  $L$ , becoming smaller in the vicinities of  $L$ , at which  $L_{pv}$  peaks. The relative relationships between  $\alpha_1$  and  $\alpha_2$  are similar to those between  $L_{v1}$  and  $L_{v2}$ .

#### 4.2 Vibroacoustic coupling with symmetric vibration mode

In this section, the (0,1) mode is taken as the symmetric vibration mode. Figure 7 shows the sound pressure levels  $L_{pv}$  and  $L_p$  as a function of the cylinder length  $L$  when thicknesses  $h = 3$  and 4 mm.  $L_{pv}$  at  $h = 3$  mm has peaks at short intervals that are shorter than those of the (1,0) mode at  $h = 4$  mm. In contrast, the peaks do not appear at equal intervals in  $L_{pv}$  at  $h = 4$  mm, observed in the entire range of  $L$ . The relationship between  $L_{pv}$  at  $h = 3$  and 4 mm is confirmed by the experimental results  $L_p$ , whose relationship between  $h = 3$  and 4 mm is similar to theoretical results.

The (0,0, $q$ ) and (0,1, $q$ ) modes are considered as the symmetry about the  $z$ -axis as well as the (0,1) mode and the (0,1, $q$ ) mode is coincident with the (0,1) mode in the radial order. The vibration and acoustic systems have the possibility for the promotion of vibroacoustic coupling because of the similarity of modal shapes

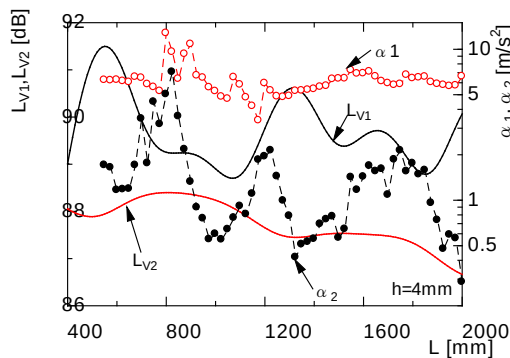


Fig. 6 Variations in theoretical level and experimental acceleration of plate vibration with cylinder length in (1,0) mode.

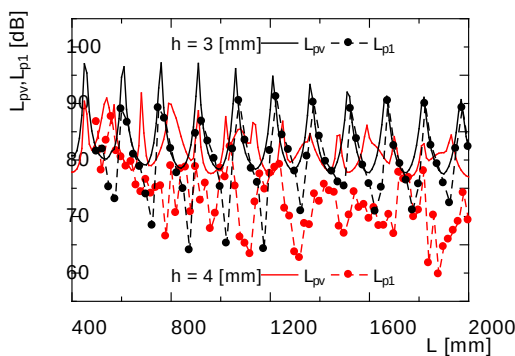


Fig. 7 Comparison between theoretical and experimental results for sound pressure level in (0,1) mode.

between the above vibration and acoustic modes. Figure 8 shows the contribution  $C_E$  of the (0,0, $q$ ) and (0,1, $q$ ) modes as a function of  $L$  with respect to  $h = 3$  and 4 mm.  $C_E$  of the (0,0, $q$ ) modes at  $h = 3$  mm occurs at the same  $L$  values as the peaks for  $L_{pv}$  and they approach 1.0. Although  $C_E$  of the (0,1,0) mode is large in the middle of the above  $L$  when  $L$  is short, they decrease gradually with increasing  $L$ . At  $h = 4$  mm, in addition to the (0,0, $q$ ) modes, several (0,1, $q$ ) modes are taken up. However,  $C_E$  of the (0,0, $q$ ) modes is close to unity as well as that at  $h = 3$  mm, while  $C_E$  of the (0,1, $q$ ) modes is scattered with changes in  $L$ .

Figures 9(a) and 9(b) show the vibration levels  $L_{v1}$  and  $L_{v2}$  as a function of  $h$ , for which  $L$  to adjust  $f_{00q}$  and  $f_{01q}$  to  $f_{01}$  is chosen, respectively. In case of taking the (0,0, $q$ ) modes, although  $L_{v1}$  is always larger than  $L_{v2}$  in the entire range of  $h$ , the minimization of both levels is confirmed at arbitrary thicknesses for each acoustic mode. However,  $L_{v2}$  is much larger in variations than  $L_{v1}$ , so that we can make sure of the gradual maximization of levels in  $L_{v2}$ . Having the same relative relationships in the vibration levels as those for the (0,0, $q$ ) modes,  $L_{v1}$  and  $L_{v2}$  for the (0,1, $q$ ) modes have many maximum and minimum values in contrast to those for the (0,0, $q$ ) modes, and their appearance is suppressed in terms of whether  $f_{01}$  reaches  $f_{01q}$ . In particular, the decided maximum and minimum values occur alternately with varying  $h$ . Thinking such a characterized value is caused by the simultaneous existence of the

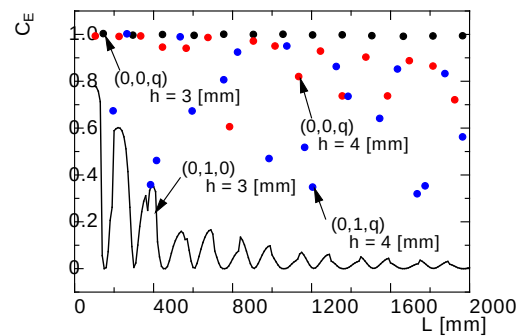
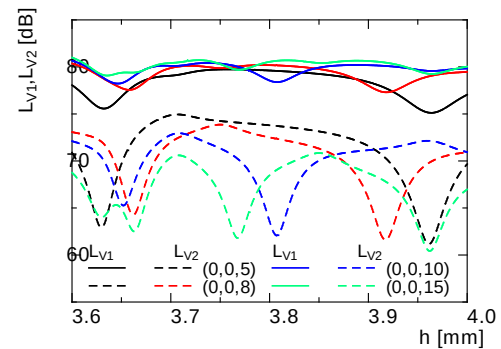
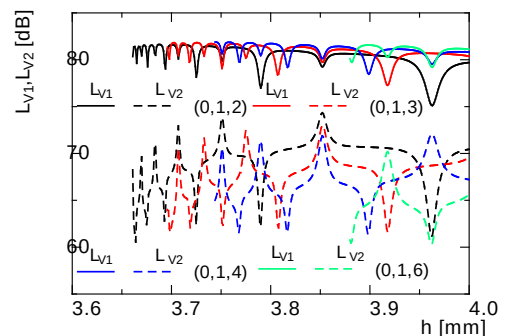


Fig. 8 Contribution of each acoustic mode to formation of sound field as function of cylinder length in (0,1) mode.



(a) Coupling with (0, 0,  $q$ ) modes



(b) Coupling with (0, 1,  $q$ ) modes

Fig. 9 Variation in plate vibration level with plate thickness.

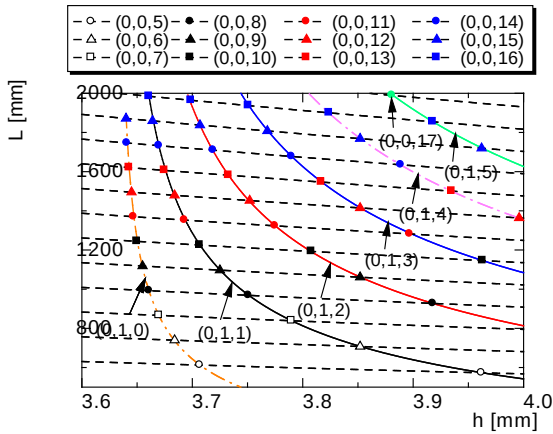


**Fig. 10** Cylinder length, at which sound pressure level is maximized by  $(0,0,q)$  and  $(0,1,q)$  modes, as function of plate thickness in  $(0,1)$  mode.

$(0,0,q)$  and  $(0,1,q)$  modes according to the results of  $C_E$ , we should consider the relationship between the  $(0,0,q)$  and  $(0,1,q)$  modes.

Figure 10 shows the specific length  $L$ , at which  $f_{00q}$  and  $f_{01q}$  are coincident with  $f_{01}$ , respectively, as a function of  $h$ .  $L$  where  $f_{00q}$  becomes identical to  $f_{01}$  decreases gradually with increasing  $h$  and varies in the longer region with increasing  $q$ . In contrast, although  $L$  where  $f_{01q}$  is identical to  $f_{01}$  decreases rapidly with increasing  $h$ , the variations are suppressed with increasing  $q$  and go across those for the  $(0,0,q)$  modes. The characterized behavior of  $L_{v2}$  takes place at the intersection points that are plotted in this figure.  $L_{v2}$  has the maximized values if the longitudinal orders of the  $(0,0,q)$  and  $(0,1,q)$  modes are identical each other in odd or even number, having the minimized values if the orders are not identical each other.

Figures 11(a) and 11(b) show  $L_{v1}$  and  $L_{v2}$  at  $h = 3$  and  $4$  mm as a function of  $L$ , respectively. The experimental accelerations  $\alpha_1$  and  $\alpha_2$  for the respective plate vibrations are also plotted in each figure.  $L_{v1}$  maintains levels and small variations in the entire range of  $L$ , while  $L_{v2}$  has the relatively large variations and tends to decrease entirely with increasing  $L$ , whereas they are close together as  $L_{pv}$  peaks, no matter what the thickness is. However,  $L_{v2}$  at  $h = 4$  mm varies irregularly with  $L$  and has significant variations in comparison with that of  $h = 3$  mm. The behaviors of  $\alpha_1$  and  $\alpha_2$  correspond to those of  $L_{v1}$  and  $L_{v2}$ , considered in the aspects as  $\alpha_1$  remains almost constant and  $\alpha_2$  decreases with increasing  $L$  while vibrating. In particular, the behavior of  $\alpha_2$  at  $h = 4$  mm is characterized as the effect of the simultaneous existence of the  $(0,0,q)$  and  $(0,1,q)$  modes. In contrast to  $\alpha_2$  at  $h = 4$  mm,  $\alpha_2$  is too small to demonstrate the theoretical results in the relationship with  $\alpha_1$  at  $h = 3$  mm. This experiment is considerably sensitive, so that several results have been obtained from a number of the excitation experiments. If the actual experiment could be brought to the situation closer to the analysis condition, we would obtain more appropriate results than the above ones.

## 5. Conclusion

In this study, the cylindrical structure with the plates at both ends was taken and one end plate was excited by the harmonic point force, and then the frequencies that cause the plate to vibrate in symmetric and unsymmetric higher modes, respectively, were employed as the excitation frequencies.

In the unsymmetric vibration mode, coupling with the fundamental acoustic mode is suppressed because of the difference in modal shapes even if the eigenfrequencies are close each other, so that the

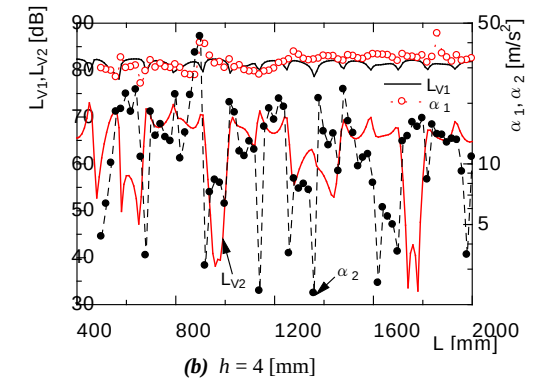
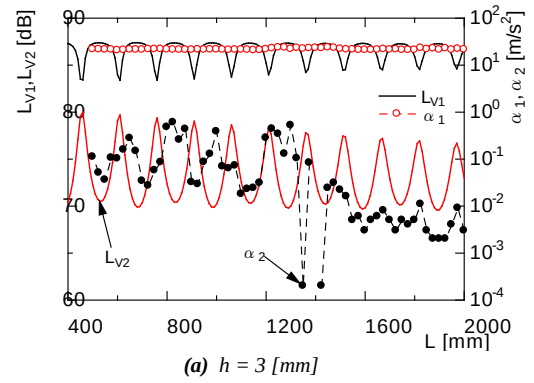


**Fig. 11** Variations in theoretical level and experimental acceleration of plate vibration with cylinder length.

vibration mode promotes coupling with a single acoustic mode that resembles it in modal shapes. The symmetric vibration mode promotes independently coupling with the fundamental acoustic mode because of the similarity of their modal shapes even if the natural frequency cannot reach the resonance frequency. As a result of the above coupling phenomena, the sound pressure level peaks regularly with the cylinder length. However, in the vibration mode whose natural frequency can reach the resonance frequency of the acoustic mode having the same radial order, the similarity of the modal shapes enables the vibration mode to couple with the acoustic mode in addition to the fundamental one, so that the sound pressure level peaks irregularly with the cylinder length. On the other hand, the vibration level of the plate not excited by the point force is smaller than that on the excitation side over the entire range of the cylinder length and the difference expands with increasing the cylinder length. However, the vibration level on non-excitation side tends to approach that on the excitation side when the sound pressure level peaks with the promotion of vibroacoustic coupling.

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